Soft Clustering of Physics Misconceptions Using a Mixed Membership Model

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ABSTRACT

Students often possess multiple, conflicting misconceptions which may be activated and expressed in different contexts. In this paper, we use a mixed membership model to explore the patterns of misconceptions in introductory physics. Mixed membership models have been widely used for modeling observations that have partial membership in several latent groups. The latent groups in the current study are misconception patterns. This model allows us to examine whether students are likely to hold a few or many misconceptions, as well as which misconceptions are likely to co-exist. Physics knowledge was measured with the Force concepts inventory (FCI). We found three dominant response patterns, with different misconceptions prominent within each pattern.

1. INTRODUCTION

Student misconceptions can be persistent, and interfere with learning unless they are addressed directly. One important characteristic of misconceptions is that students possess many different knowledge components simultaneously, so that the particular schema or rule a student uses to solve a question depends on many different factors, including the context of the question [4]. This paper presents a case-study for using a mixed-membership model [1] to capture the characteristics and coherent patterns among students' misconceptions in introductory physics. Mixed membership model allows students to possess different misconception patterns (profile) across test questions. In this study, we focus on two questions: (1) What are the common misconception pattern students possess across the test, and which misconceptions tend to co-occur. (2) How much does each student exhibit each pattern?

2. METHODS

2.1 Mixed membership model

Mixed membership models allow an individual to switch profiles across contexts, test items. How much each individual uses each profile is parametrized by $\theta_i = (\theta_{i1}, \dots, \theta_{iK})$. The components of θ_i are nonnegative and sum up to 1. Z_{ij} indicates the profile that student i uses for item j, so that

$$Z_{ij}|\theta_i \sim Multinomial(\theta_i).$$

Each latent profile has its own probability distribution for observed variables. Since the items from the case study are multiple choice, if X_{ij} denotes the observed response for student i on item j, then $X_{ij}|Z_{ij} = k \sim Multinomial(\beta_{(j|Z_{ij}=k)})$, where $\beta_{(j|Z_{ij}=k)} = (\beta_{kj1}, \dots, \beta_{kjm}, \dots, \beta_{kjM})$, β_{kjm} denotes the probability that a student using profile k on item j will select option m, and M is the number of options.

In the mixed membership model, the generative process is given by [5,6]:

1. For each item $j = 1, \dots, J$, draw $\beta_{(j|Z=k)} \sim Dirichlet(\eta)$, for $k = 1, \dots, K$.
2. For each individual $i = 1, \dots, N$
   (a) Draw $\theta_i \sim Dirichlet(\alpha)$
   (b) For each item $j = 1, \dots, J$,
      i. Draw $Z_{ij}|\theta_i \sim Multinomial(\theta_i)$.
      ii. Draw $X_{ij}|Z_{ij} \sim Multinomial(\beta_{(j|Z_{ij}=k)})$,

Here η and α are prior parameters. These could be estimated in an empirical-Bayes fashion. We choose to set these parameters to incorporate prior information, and stabilize the model.

2.2 FCI Data

From 1995-1999, 4450 high school students responded to The Force Concept Inventory (FCI), one of the most commonly used assessments in physics to measure students' understanding of concepts on Newtonian mechanics. We foucsed on the pre-test scores from a larger study [3]. The FCI consists of 30 multiple-choice items, with 18 items measuring *Newton's Second Law.* Most of the distractor options on this test were designed to map to a common physics misconception, though some distractors are statements that cannot be explained by physics theories. More detailed explanation of these misconceptions can be found in [2].

3. RESULTS

We estimated the mixed membership model using MCMC with 5,000 iterations (1,000 burn-in). We placed a weakly informative prior on $\beta_{(j|Z=1)}$, of $\eta_{j1} = (50, 1, 1, 1, 1)$, and a flat prior to all the other parameters.

3.1 Number of Profiles

We fit mixed membership model with three to seven profiles. The same misconceptions were found to co-exist regardless of the number of profiles. In the 3-profile model, students have the most distinct probabilities of selecting a particular response across profiles, and were more likely to exclusively belong to one of the profiles ($\theta_{ik} > 0.8$). Thus, we can say that three profiles is representative of students' misconception patterns and in this paper, we focus on the 3-profile model.

3.2 Students' Membership in the Profiles

Profile memberhsip of each student is captured by the parameter $\theta_i = (\theta_{i1}, \theta_{i2}, \theta_{i3})$ shown in Figure 1. The proportion of students who exclusively belong to profile 3 is the highest, followed by profile 2 and profile 1. There are many students who are between profile 2 and profile 3 as well as between profile 3 and profile 1. Far fewer students fall between profile 2 and profile 1.

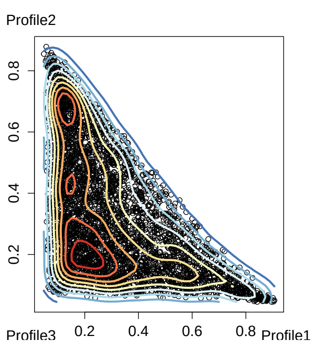

Figure 1: Contour map of posterior distribution for students' membership in the three profiles. X nad Y axes represent θ_{i1} for Profile 1 and Profile 2 (θ_{i1}) respectively. Profile 3 can be obtained by $\theta_{i3} = 1 = \theta_{1i} - \theta_{i2}$

3.3 Characteristics of Profiles

Each profile is parameterized by a probability distribution over the responses to each item, $\beta_{(j|Z=k)} = (\beta_{kj1}, \ldots, \beta_{kj5})$. We illustrate the characteristics of each profile using items that measure Newton's Second Law of Motion, and these characteristics hold up for all the items in the FCI instrument.

Misconception Profile (profile 3) This profile is characterized by high probability on responses containing misconceptions. Recall also, that this profile had the most students that belonged to it exclusively, as well as large numbers of students who were between it and the other profiles (Figure 1). In this profile, some misconceptions, such as *impetus dissipation* are observed repeatedly across items. However, we also observe that the activation of a misconception depends on items. For example, the misconception *impetus supplied by "hit"* is likely to be observed in item 30 even though it is also associated with item 11. This profile has the most profound implications for instruction since it is the largest, and demonstrates that students tend to not hold a single misconception, but rather many misconceptions that co-exist and may be expressed in different contexts.

Mostly Correct Profile (profile 1). This profile places a high probability on the correct response for most items, and has the smallest number of students that have high membership in the profile. However, on a few items, this profile is also associated with misconceptions. Some of these misconceptions, such as *largest force determines motion* were shared by the other profiles which instructors will want to address, and some of them tend to be of a higher-level.

Uniform Profile (profile 2). In general, the probability of choosing an option was similar across at least three options for most of the items. This profile has a large number of students who belong almost exclusively to it. Even when we increased the number of profiles, it did not disappear, nor decompose into separate profiles. These observations indicate that students in this profile do not have any coherent pattern in their responses.

4. CONCLUSION AND DISCUSSION

This study illustrates how mixed membership models can be a good tool to summarize a number of misconceptions into fewer numbers of profiles by identifying misconceptions that are likely to co-exist. Among the three profiles we found with FCI data, the majority of students had partial or complete membership in the *misconception profile*. The high coherence of co-existing misconceptions across a large number of students in this profile demonstrates the real power of this mixed membership analysis. By finding coherent patterns exhibited by many students at least some of the time, we find evidence that may suggest new theory. Future work can focus on the challenge of deciding an optimal number of profiles when conducting mixed membership models and the assumption that Z_{ij} depends on both i and j. Profile transitions between pre- and post-test should also be examined.

5. REFERENCES

[1] E. Erosheva, S. Fienberg, and J. Lafferty. Mixed-membership models of scientific publications. *Proceedings of the National Academy of Sciences*, 101(suppl 1):5220–5227, 2004.

[2] D. Hestenes and J. Jackson.

[3] D. Hestenes, M. Wells, G. Swackhamer, et al. Force concept inventory. *The physics teacher*, 30(3):141–158, 1992.

[4] K. R. Koedinger, A. T. Corbett, and C. Perfetti. The knowledge-learning-instruction framework: Bridging the science-practice chasm to enhance robust student learning. *Cognitive science*, 36(5):757–798, 2012.