Learning Curves for Problems with Multiple Knowledge Components

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- Intelligent Tutor Systems (ITS): high level of interactivity, natural to associate 1 knowledge component (KC) per step.
- In many homework systems, students just input the final answer, which typically depends on many KCs.

→ Assignment of blame problem

Question: Can we, in principle, untangle the KCs?
Answer: yes, with a careful error analysis

The model

- Simplest possible:
  - $P_{t,k}$ is the probability that a student will apply KC k correctly on opportunity t.
  - Use the set $\{P_{t,k}\}$ as the model parameters.
- The log likelihood is

$$\log(\mathcal{L}) = \underbrace{\sum_{s,p \in \mathcal{C}_s} \log(\xi_{s,p})}_{\text{correct}} + \underbrace{\sum_{s,p \in \mathcal{I}_s} \log(1 - \xi_{s,p})}_{\text{incorrect}}$$

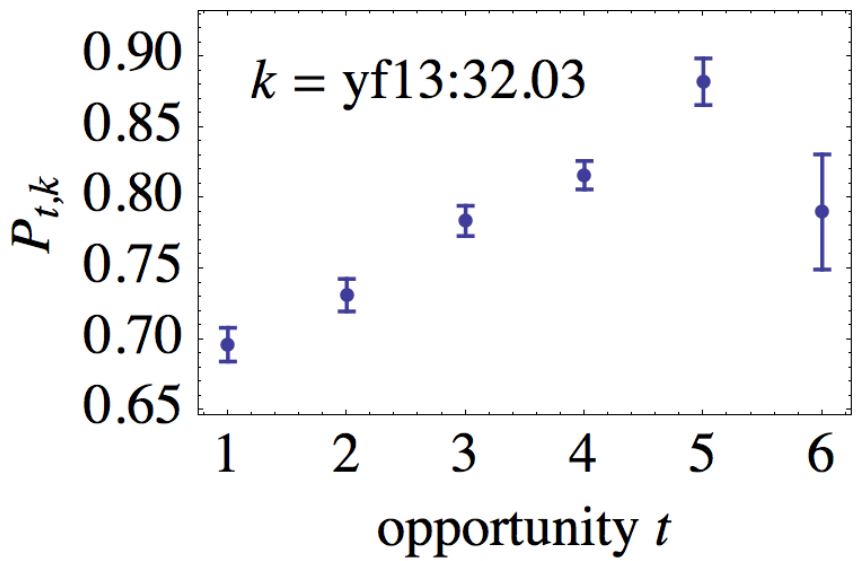
where $\xi_{s,k}$ is the model-predicted probability that student s will get problem p correct. It is the product of probabilities for KCs associated with that problem:

$$\xi_{s,p} = \prod_{t,k \in \mathcal{T}_{s,p}} P_{t,k}$$

We assume a conjunctive model: student must apply all KCs correctly to solve the problem.

$$\xi_{s,p} = \prod_{t,k \in \mathcal{T}_{s,p}} P_{t,k}$$

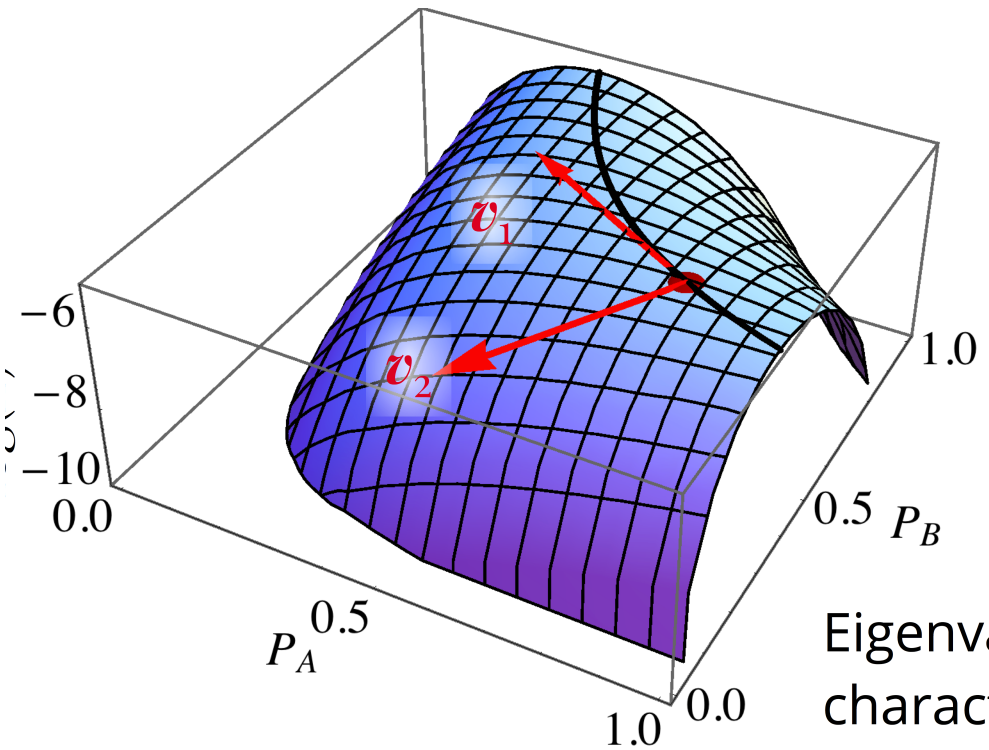
The case of one KC per problem and fitting to student data gives the usual learning curves.



Multiple KCs

Simple example: 1 problem with KCs A and B, 1 opportunity

$$\log(\mathcal{L}) = N_{\text{correct}} \log(P_A P_B) + N_{\text{incorrect}} \log(1 - P_A P_B)$$



- Maximum is along a line.
- Numerical maximization routine will find a point.
- Find the Hessian matrix at that point:

$$\mathbf{H}_{k,l} = \frac{\partial^2 \log(\mathcal{L})}{\partial P_k \partial P_l}$$

Eigenvalues and eigenvectors of **H** characterize maximum:

v_1 has eigenvalue $\lambda_1 = 0$ ⟵ nullspace

v_2 has eigenvalue $\lambda_1 < 0$

Remove 1 KC that overlaps nullspace.

General procedure

- Find the maximum likelihood point.
- Calculate the Hessian matrix $\mathbf{H}$.
- Find the associated eigenvalues and eigenvectors.
- Choose a cutoff for small eigenvalues İ nullspace of $\mathbf{H}$ with n eigenvectors
- Find n KCs having the largest overlap with the nullspace of $\mathbf{H}$ and remove them.
- The new Hessian matrix $\mathbf{H}'$ will be invertible.
- The inverse $-(\mathbf{H}')^{-1}$ is an estimator of the standard covariance matrix.

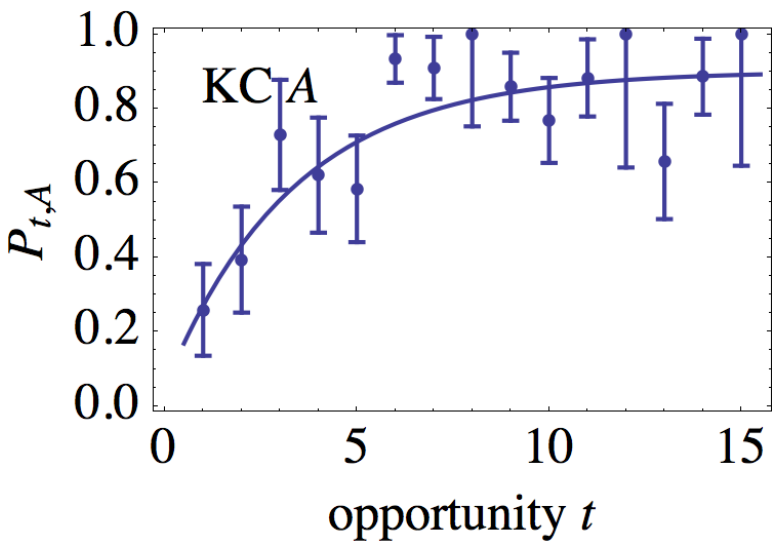
➡ Model parameters that are poorly determined will have large standard errors.

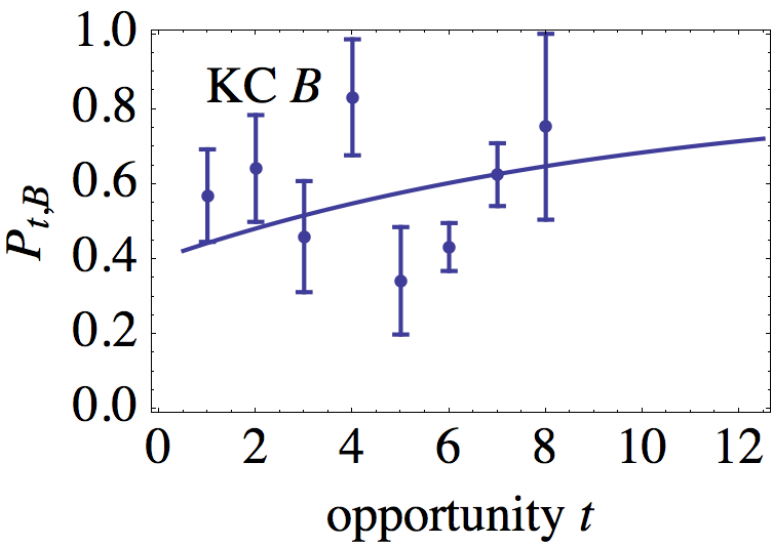
Example

- 20 simulated students solve 15 problems in random order
- KC content of problems (note that KC B never appears alone)

A	A	A	AB	AB
A	A	AB	AB	AB
A	A	AB	AB	AB

Using student data, calculate maximum likelihood and errors.





Conclusion

One can solve the assignment of blame problem if

- Students in population solve problems in different orders (or different problems)
- Problems have varying KC combinations

Careful error analysis needed to determine level of success.

Two-step process:

- Remove problematic KCs
- Look at standard errors & covariance matrix