

# Soft Clustering of Physics Misconceptions Using a Mixed Membership Model



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## Characteristics of Misconceptions

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- Misconceptions are persistent and can impede students' learning unless they are addressed.
- Misconceptions are context dependent [1]:
  - Students may overcome a misconception in one test question and fail in another for the same misconception.
- A group of students may share common misconceptions. [2]

## Questions

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- What are the patterns in students' misconceptions (profiles)? Which misconceptions tend to co-occur?
- Can a mixed membership model capture these patterns?

## Mixed Membership Model

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To model the characteristics of misconceptions. Students may belong to different profiles in different test items.

- $Z_{ij}$ : the profile student  $i$  belongs to for item  $j$ .
- $\theta_i = (\theta_{i1}, \dots, \theta_{iK})$ : the probability that student  $i$  belongs to profile  $k$ .

$$Z_{ij} | \theta_i \sim \text{Multinomial}(\theta_i)$$

$$\theta_i \sim \text{Dirichlet}(\alpha)$$

- $X_{ij}$ : the option student  $i$  chooses for item  $j$ .
- $\beta_{kj} = (\beta_{kj1}, \dots, \beta_{kjm})$ : given profile  $k$ , the probability that students choose option  $m$  in item  $j$ .

$$X_{ij} | Z_{ij} \sim \text{Multinomial}(\beta_{kj})$$

$$\beta_{kj} \sim \text{Dirichlet}(\eta)$$

## Data

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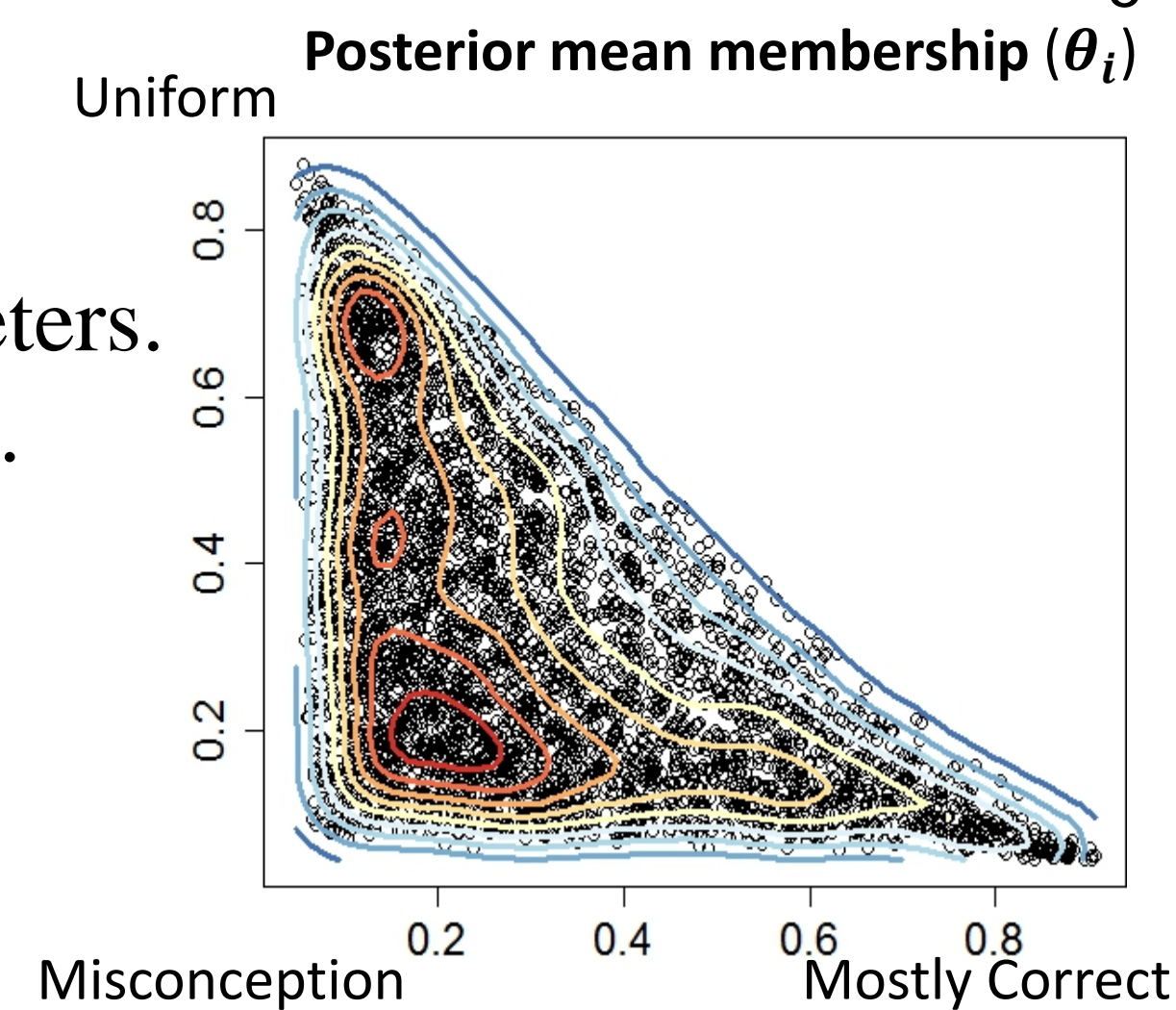
- 4450 high school students, enrolled from 1995- 1999
- Force Concept Inventory
  - Includes 30 multiple-choice items.
  - Most distractors map to a specific physics misconception.

## Results

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### Estimation

- Employed MCMC to estimate parameters.
- Compared models with 3 to 7 profiles.
- Focus on 3-profile model, because conclusions based on all models were similar.



### Overall patterns of the profiles

- Mostly Correct Profile, Uniform Profile, Misconception Profile
- 2 example items illustrate the overall patterns of the 3 profiles:

Item 1:

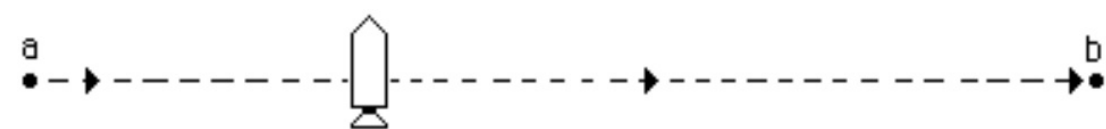
Two metal balls are the same size but one weighs twice as much as the other. The balls are dropped from the roof of a single story building at the same instant of time. The time it takes the balls to reach the ground below will be:  
 (A) about half as long for the heavier ball as for the lighter one.  
 (B) about half as long for the lighter ball as for the heavier one.  
 (C) about the same for both balls.  
 (D) considerably less for the heavier ball, but not necessarily half as long.  
 (E) considerably less for the lighter ball, but not necessarily half as long.

Posterior mean response probabilities ( $\beta$ ) for Item 1.

	Mostly Correct	Uniform	Misconception
A	.004	.155	.128
B	.003	.218	.016
C *	.983	.278	.379
D	.007	.236	.408
E	.004	.112	.068

Item 22:

A rocket drifts sideways in outer space from point "a" to point "b" as shown below. The rocket is subject to no outside forces. Starting at position "b", the rocket's engine is turned on and produces a constant thrust (force on the rocket) at right angles to the line "ab". The constant thrust is maintained until the rocket reaches a point "c" in space.



As the rocket moves from position "b" to position "c" its speed is:

- (A) constant.
- (B) continuously increasing.
- (C) continuously decreasing.
- (D) increasing for a while and constant thereafter.
- (E) constant for a while and decreasing thereafter.

Posterior mean response probabilities ( $\beta$ ) for Item 22.

	Mostly Correct	Uniform	Misconception
A	.248	.174	.316
B*	.532	.344	.099
C	.003	.170	.007
D	.214	.176	.538
E	.002	.139	.040

## Co-Occurring Misconceptions

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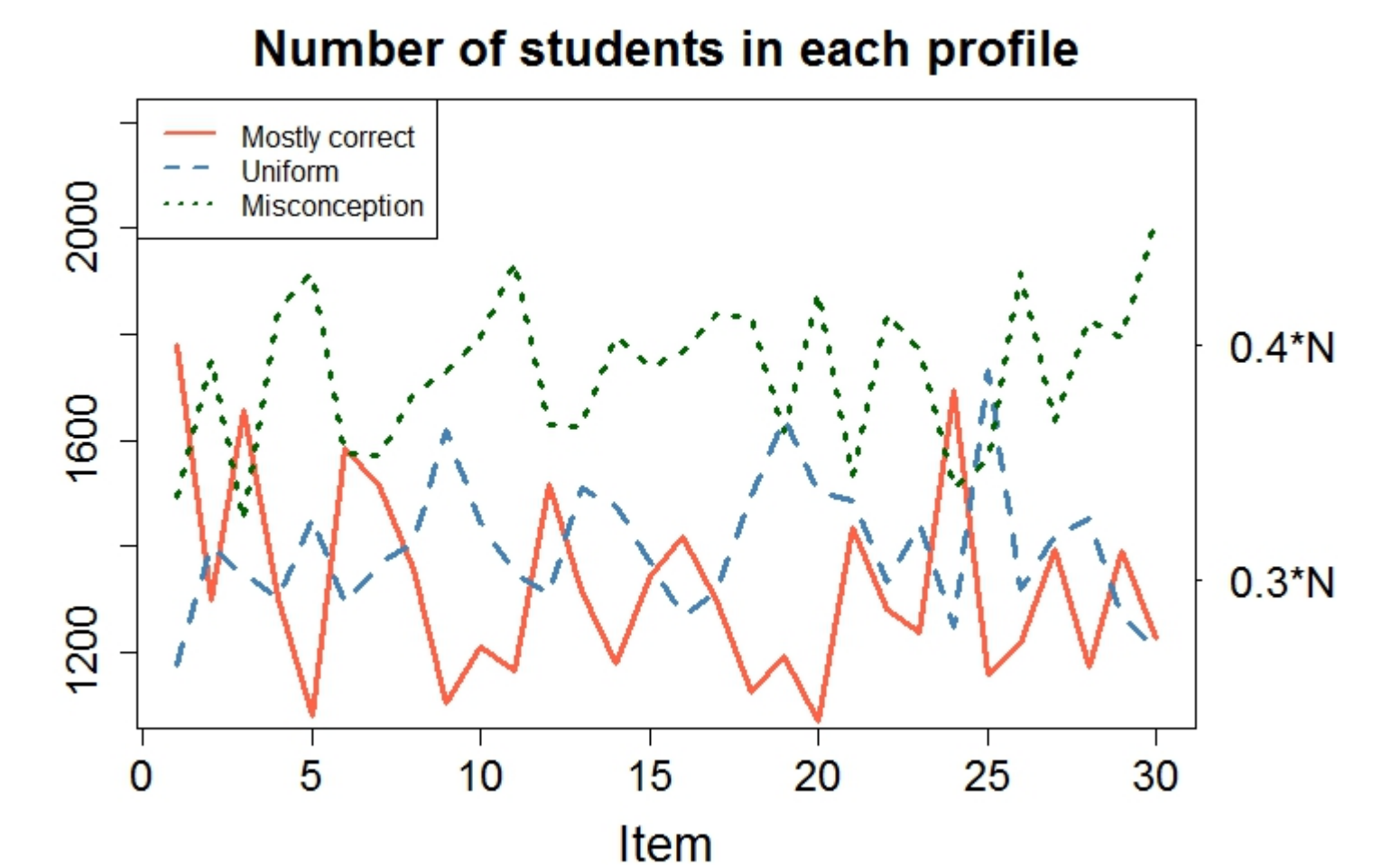
- Misconception Profile: a set of misconceptions co-exist among a large number of students.

Misconceptions		Misconceptions	
Item 1	heavier objects fall faster	Item 24	impetus dissipation
Item 13	impetus dissipation	Item 27	motion implies active force
Item 22	force causes acceleration to terminal velocity	Item 30	impetus supplied by "hit"
...	...	...	...

## Limitations of Mixed Membership

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- The type of profiles and the individual probability of belonging to each profile do not vary with item.



- Uniform profile may be absorbing variance for students that do not match strongly shared patterns.

## Conclusions

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- Students don't have isolated misconceptions.
  - Many misconceptions can be explained by "no distinction between force & momentum".

## Reference

- [1] Koedinger, K. R., Corbett, A. T., & Perfetti, C. 2012. The knowledge-learning-instruction framework: bridging the science-practice chasm to enhance robust student learning. Online Submission.
- [2] Huang, C. and Mislevy, R.J., 2010. An application of the polytomous Rasch model to mixed strategies. Handbook of polytomous item response theory models, pp.211-228.