Uncovering Class-Wide Patterns in Responses to True/False Questions

Andrew Pawl
Department of Engineering Physics
University of Wisconsin-Platteville
Platteville, WI 53818
pawla@uwplatt.edu

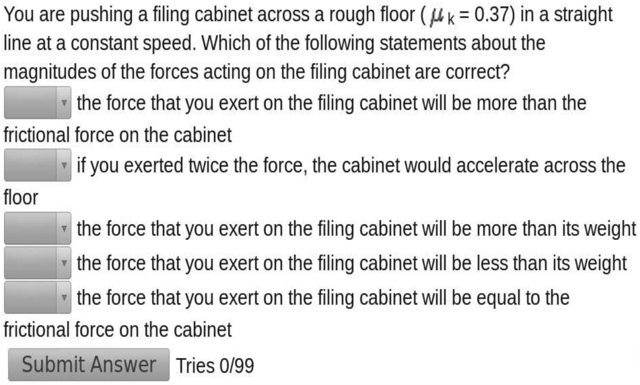
ABSTRACT

A popular type of problem in online homework involves a set of several true/false statements and requires that students submit their answers to all the statements at once. Such problems can force a student to submit many responses to the same true/false statement. It is possible to examine student submission patterns to problems of this type with the goal of determining which of the individual true/false statements exhibit a large proportion of response switches and which statements exhibit largely consistent responses. This paper describes algorithms that allow an instructor to uncover those statements that exhibit class-wide randomness and also those that exhibit a class-wide preference for an incorrect response. The utility of the approach is suggested by the fact that examining statements which emerge as outliers according to these metrics uncovers several statements that probe known student misconceptions.

1. INTRODUCTION

A popular type of problem in the LON-CAPA online homework network [1] consists of a situation or set of situations followed by five related true/false statements [2] (an example is shown in Fig. 1). The student is required to submit answers to all the true/false statements at once and receives only correct/incorrect feedback. A student who submits an incorrect answer will not know which of the statements has been answered incorrectly or even how many of the statements are incorrect, and so may submit as many as $2^5 = 32$ responses before arriving at the correct answer.

One goal of this work is to develop a means to detect statements to which the class consistently responds incorrectly. Such response patterns correlate with *strong misconceptions.* The definition of "strong misconception" in the context of this work is an intuitive belief that is in conflict with the concepts taught in the course. An example is the first statement in the problem shown in Fig. 1. Research has shown that students in introductory physics courses have a strong tendency to believe that there must be a net force in the direction of motion, even when this belief conflicts with Newton's First Law [3, 4]. Thus, one might expect the class to consistently answer "True" to this statement, even when forced to answer multiple times.


You are pushing a filing cabinet across a rough floor (μ_k = 0.37) in a straight line at a constant speed. Which of the following statements about the magnitudes of the forces acting on the filing cabinet are correct?

the force that you exert on the filing cabinet will be more than the frictional force on the cabinet

if you exerted twice the force, the cabinet would accelerate across the floor

the force that you exert on the filing cabinet will be more than its weight

the force that you exert on the filing cabinet will be less than its weight

the force that you exert on the filing cabinet will be equal to the frictional force on the cabinet

Figure 1: An example of the type of problem discussed in this paper.

Another complementary goal is to investigate whether significant class-wide randomness in the answers to a given statement can be an indicator of *incomplete understanding.* Again, the problem of Fig. 1 provides a useful illustration. If a significant portion of the class is indeed convinced that a net force is necessary to produce constant velocity, this could produce a conflict in the minds of the students about the consequences of applying more force. Will the extra force produce a steady acceleration in accordance with Newton's Second Law, or will there be a transient acceleration dropping to zero when the appropriate velocity is reached? Because of these conflicting ideas, one might expect students to exhibit a tendency to change their answer to the second statement shown in Fig. 1.

2. ASSESSING CONSISTENCY

A class with an average near 100% correct submissions to a certain statement is consistently giving the correct response. However, because a true/false statement has only one incorrect response, an average near 0% correct submissions *also* implies consistent responses (the class is continuing to respond with the one incorrect answer). If the class is answering randomly, the average will approach 50% correct submissions as the number of tries becomes large. Thus, a submission-weighted consistency score C_{sw} can be defined:

$$C_{sw} = \frac{N_{s,correct}}{N_{s,tot}} - 0.5 \tag{1}$$

where $N_{s,correct}$ is the number of correct submissions to the statement and $N_{s,tot}$ is the total number of submissions to the statement. With this definition, $C_{sw} = +0.5$ is complete correctness and $C_{sw} = -0.5$ is complete incorrectness.

A respondent-weighted measure of the consistency of a class on a particular statement C_{rw} can be defined by analyzing the first submission of each respondent. As an equation:

$$C_{rw} = \frac{N_{s,i,correct} - N_{s,i,incorrect}}{N_{r,tot}} \tag{2}$$

where $N_{s,i,correct}$ is the number of correct initial submissions, $N_{s,i,incorrect}$ is the number of incorrect initial submissions and $N_{r,tot}$ is the total number respondents.

The overall consistency score C_{tot} is then defined:

$$C_{tot} = C_{sw} \times C_{rw} \times (\mathrm{sign}(C_{sw}) + \mathrm{sign}(C_{rw})). \tag{3}$$

3. ASSESSING RANDOMNESS

The second goal is to uncover statements that produce frequent switching of the response. The total number of possible switches for a class making $N_{s,tot}$ submissions to a statement is $N_{s,tot} - N_{r,tot}$ where $N_{r,tot}$ is the number of respondents. A switch is "realized" if the current submission is different from the prior one. A submission-weighted randomness score R_{sw} can be defined as the fraction of possible switches that are realized:

$$R_{sw} = \frac{N_{s,switch}}{N_{s,tot} - N_{r,tot}} \tag{4}$$

where $N_{s,switch}$ is the number of submissions that represent a switch of the answer from the immediate predecessor.

A respondent-weighted measure of randomness R_{rw} can be defined by determining what fraction of the students who responded to the statement ever switched their response from correct to incorrect. As an equation:

$$R_{rw} = \frac{N_{r,correct\rightarrow incorrect}}{N_{r,tot}} \tag{5}$$

where $N_{r,correct\rightarrow incorrect}$ is the number of respondents who ever switched their response from correct to incorrect.

The overall randomness score is defined:

$$R_{tot} = R_{sw} \times R_{rw}. \tag{6}$$

4. EVIDENCE FOR VALIDITY

Fig. 2 is a scatter plot of the overall consistency score versus the overall randomness score for 250 true/false statements from problems of the type described. The plot allows a reexamination of the example of Fig. 1. The expectation outlined in the Introduction is that the first statement shown in the problem of Fig. 1 should qualify as a strong misconception and that the second statement should exhibit significant randomness. The two points outlined with circles in Fig. 2 correspond to these statements. As expected, the first statement has a strongly negative consistency score (-0.33) while the second statement has a strong randomness score (0.25).

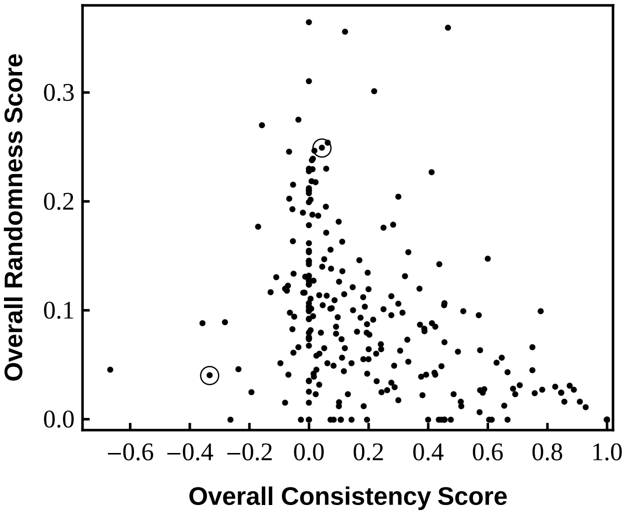


Figure 2: R_{tot} (defined in Eq. 6) vs. C_{tot} (defined in Eq. 3) for 250 true/false statements.

5. CONCLUSIONS

This paper has presented data-mining algorithms for assessing the consistency and the randomness of student responses to individual true/false statements. These algorithms are directly applicable to problems involving several linked true/false statements, which have been implemented in online homework. Investigation of examples indicates that the consistency score can uncover class-wide misconceptions and the randomness score can be a useful indicator of incomplete understanding among the class. Both scores can also serve to uncover errors in problem construction. The promise of the approach is that a simple question format that is suitable for use in online homework or as part of online courses can uncover the specific concepts that give a significant portion of the class problems.

6. ACKNOWLEDGMENTS

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7. REFERENCES

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