

# Discovering the Structure of Mathematical Problem Solving

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## ABSTRACT

It is possible to combine multivariate pattern analysis (MVPA) and hidden Markov models (HMM) to discover the major phases that students go through in solving complex problems. I will illustrate this methodology by applying it to the learning of graphical isomorphs of algebra problems. We discovered a sequence of 5 major phases that students went through: An Orient Phase where they identified the problem to be solved, an Encode Phase where they encoded the needed information, a Compute Phase where they performed the necessary arithmetic calculations, a Transform Phase where they performed any mathematical transformations, and a Respond Phase where they generated the answer. The duration of the Compute and Transform Phases were they only ones that distinguished different problem types. Increased duration in these two phases is also associated with making errors. Looking at learning, 2 features distinguished the problems on which participants came to understand a new problem type. First, the duration of late phases of the problem solution increased. Second, there was increased activation in the rostrolateral prefrontal cortex (RLPFC) and angular gyrus (AG), regions associated with metacognition. We think this indicates the importance of reflection to successful learning.

## SHORT BIO

John Anderson received his B.A. from the University of British Columbia in 1968 and his Ph.D. from Stanford University 1972. He has been at Carnegie Mellon University since 1978 where he is the Richard King Mellon Professor of Psychology and Computer Science. He has been served as president of the Cognitive Science Society, and has been elected to the American Academy of Arts and Sciences, the National Academy of Sciences, and the American Philosophical Society. He is the current editor of *Psychological Review*. He has received numerous scientific awards including the American Psychological Association's Distinguished Scientific Career Award, the David E. Rumelhart Prize for Contributions to the Formal Analysis of Human Cognition, the inaugural Dr A.H. Heineken Prize for Cognitive Science, and the Benjamin Franklin Medal in Computer and Cognitive Science.

He is known for developing ACT-R, which is the most widely used cognitive architecture in cognitive science. Anderson was also an early leader in research on intelligent tutoring systems. Computer systems based on his cognitive tutors teach currently mathematics to over 500,000 children in American schools. He has published a number of books including *Human Associative Memory* (1973 with Gordon Bower), *Language, Memory, and Thought* (1976), *The Architecture of Cognition* (1983), *The Adaptive Character of Thought* (1990), *Rules of the Mind* (1993), and *The Atomic Components of Thought* (1998 with Christian Lebiere), and *How Can the Human Mind Occur in the Physical Universe?* (2007). His current research interest is focused on combining cognitive modeling and brain imaging to understand the processes of mathematical learning.