Exploring the Impact of Spacing in Mathematics Learning through Data Mining

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ABSTRACT
Laboratory studies suggest that long term retention of Mathematics learning is enhanced by spaced, as opposed to massed, practice. However, little evidence has been evinced to demonstrate that such spaced learning has a positive impact in real world learning environments, at least partly because of entrenched pedagogy and practice, whereby students are encouraged to engage with Mathematics in a very sequential manner - thus leading to massed learning episodes. Indeed, much educational practice and the structure of Mathematics textbooks lend themselves to massed rather than spaced learning. However, in online learning such spaced practice is possible and more practically achieved. Predicting learner outcomes from data in a popular online Mathematics learning site shows that in this data set spacing seems to have a negative effect on retention at a later time.

Keywords
Mathematics, spaced learning, learning science, online learning

1. INTRODUCTION
Learning efficiently is one of the main drivers of personalized instruction. By ensuring that students engage with material only for as long as they need to in order to master it, intelligent instruction can push students further in less time, allowing outcomes to be improved more rapidly, and also to reduce the risk of boredom and loss of motivation. In addition, retention over longer time scales is important to the goals of Education as a whole. While the old adage "Education is what is left once what is learned has been forgotten" is oft quoted, in many Educational contexts, and in particular Mathematics, the necessity of prerequisite knowledge for learning higher order material means that such forgetting is far less desirable.

Until relatively recently in pedagogical practice (as shown by the design of Mathematics textbooks), it was thought that the most efficient way for a student to learn Mathematics in a way that facilitated later retrieval was *overlearning* - the continued practice of a procedure after mastery has been achieved. This *massed* (as opposed to *spaced*) practice model explains the design of Mathematics textbooks, where, by chapter, exercises are massed by a small number of procedures that need to be applied. By contrast, a spaced learning methodology would require intermingled exercises requiring application of different kinds of procedure, but with procedures recurring multiple times over several study sessions.

Spacing has been a core component of recent advances in our understanding of the Science of Learning. Rohrer and Pashler[7], drawing on work by Rohrer and Taylor[8], identify the empirical support for using such spaced learning episodes in the learning of Mathematics. Rickard et al.[6] examined the role of spacing in promoting retrieval over calculation in mathematics, and spacing of learning has been assessed in the college Mathematics classroom by Butler and colleagues[1]. Both found spacing to have positive effects on Mathematics learning. However, most recent work has focused more on the effect of spacing on declarative fact learning, with much of the successful practical application focused on foreign language vocabulary learning[4][5][9]. If these techniques can be extended to Mathematics learning, then considerable learning gains could be achieved.

Such hypotheses are best tested through a more controlled manipulation of the spacing regime - in the online learning context, using an A/B test common in most website implementations. Exposing some subset of users to a spaced learning regime, while recommending massed learning to the remainder. However, it is also possible to examine the impact of spaced learning in a somewhat more confounded way by looking at spaced learning that has occurred naturally during the course of student engagement.

2. DATA
The data being analysed are logs from Khan Academy's interactive Mathematics exercise platform. Students answer exercises, and are given instant feedback. The data recorded for each attempt includes the exercise type, the instance of the exercise, the answers given by the student, the time the student spent on the page while answering, the time it was attempted, and whether the student used a hint or not.

2.1 Spaced Learning

Khan Academy has attempted to implement spaced learning within its site design mostly derived from the spaced repetition algorithm popularized by Leitner[3]. In the Leitner System cards that have been correctly memorized are pushed back into a later set, whereas incorrectly answered cards are placed into the first set. The first set is reviewed on every cycle, with each set beyond being reviewed one less time per cycle (for N boxes, a cycle will consist of N review sessions).

The variable implementation of this spacing design over time in the Khan Academy site (including the use of A/B testing for various implementations of this spaced repetition algorithm), in addition to the voluntary engagement with the software by student users has served to create a data set with a large variety of spacing schemes (although somewhat confounded by other variables). Using this data, we are conducting a post hoc analysis of spaced versus massed practice. This will help to shed light on the impact of spaced repetition on learning of particular Mathematics skills.

3. ANALYSIS

Recent experimental studies on spaced learning have generally been constructed around one or more temporally separated (by periods of more than a day) study sessions, followed by a further temporally separated recall session, where retention of what has been learned is measured[2]. In order to emulate this design for each student, data, subdivided by exercise, were separated into study sessions (any gaps of a day or more were assumed to constitute a separate study session). In order to have an outcome measure by which to measure student learning, the final session was taken to be the retention session.

3.1 Data Selection

In order to ensure more meaningful comparisons, all student/exercise pairings with only one session associated with them (and therefore no differentiable outcome measure) were discarded, as were students who had made less than ten attempts across all sessions on that particular exercise. A random subsample was chosen for analysis, with data from 13528 students, and a total of 155602 student/exercise pairs. All data were normalized before fitting in order to render model coefficients more meaningful.

4. RESULTS

In order to assess the potential contribution of the effect of spacing, a logistic regression model using L2 regularization (strength parameter set by 10-fold cross validation) was fitted to predict student performance during the retention session. The independent variables included in the model were: mean accuracy across all study sessions, mean accuracy in the most recent study session, total time spent on exercises during study sessions, total number of study sessions, and total number of attempts during study sessions. While the model performed relatively poorly, (achieving approximately 58% accuracy on the test data) similar performance was seen predicting from most subsets of the independent variables. Only total time spent failed to lend any power to the model.

Table 1: Coefficients for Normalized Variables

Mean Study Accuracy	38.25
Recent Accuracy	-15.61
Total Study Time	0.74
Number of Study Sessions	-13.36
Study Attempts	8.59

5. CONCLUSIONS

The results seem to indicate that, at least in the case of the Khan Academy data, that spaced learning does not help with later retention. However, as much of the engagement takes place over relatively short time scales (with the median interval between study and retention being ten days). Further analysis will look at the impact of spaced learning not only on later retention of that skill, but also on learning skills for which the learned skill is a prerequisite. This will allow the impact of spaced learning to be assessed absent the compressed nature of engagement with individual exercises.

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