coefficients significantly predict each group's post-test outcomes, controlling for pre-test.

Despite the focus of the AFM+GroupRate model on student-level differences, adding the per-group rate parameter produces more accurate estimates of KC parameters, based on the model's superior performance in student-stratified CV for the vast majority of datasets. The only information the model gets for fitting test data in student-stratified CV ("unseen" students whom the model has no information about with respect to ability, learning rate, or group) are the KC parameters. For this reason, AFM+GroupRate may be useful for data-driven refinement of KC parameters, which in turn has implications for instruction (e.g., parameter-setting in Knowledge Tracing based cognitive tutors [14]).

Compared to other statistical models extending AFM (Performance Factors Analysis [12], Instructional Factors Analysis [5], Recent Performance Factors Analysis [7]), AFM+GroupRate adds relatively few parameters (only three) to AFM but achieves consistent and substantive improvements in prediction. These three parameters' coefficient estimates are consistently interpretable (the per-group learning rates are ordered according to intuitions about each group's learning curve steepness), and the model avoids overloading on the interpretation of parameters.

We conducted extensive post-hoc analyses to interpret what the three learning groups actually reveal about student behavior and did not find evidence that the groups detect learning speed as an inherent trait, per se. For example, high ability students did not tend to be in the "steep" group, and low ability students did not tend to be in the "flat" group. Rather, the amount of improvement per opportunity seems to differ, more generally, depending on where the learner is on his/her *true* learning curve for any given skill. That is, the improvement per opportunity may be different for the earliest opportunities on a skill than for much later opportunities on a skill. Different students' learning curves within cognitive tutor data may vary because they start using the cognitive tutor at different points of their true learning curves for any given skill, depending on their experience with that skill prior to tutor use. We found evidence supporting this notion in post-hoc analyses. Considered in conjunction with the lack of evidence for a per-student learning rate, our findings contradict the intuitive notion that some students naturally learn faster than others.

5.2 Limitations and future work

The present results somewhat conflict with a finding from [15] that adding a per-student learning rate parameter to BKT yields substantial improvements in model fit, though we note that that report did not provide an interpretation nor any external validity evidence. We did not observe a benefit when adding a per-student learning rate parameter to AFM. Further work to compare these per-student parameter estimates across AFM and BKT and to externally validate the estimates from individualized BKT will provide insight into this issue.

Based on our post-hoc analyses, classification into the "flat/declining" group seems to capture high-ability students who descend into noisy performance at late opportunity counts (indicating boredom and/or "gaming the system" [2]) and low-ability students who never seem to improve ("wheel spinners" [1]). It would be interesting to validate this by seeing whether the detectors in [1] and [2] yield the same students when tested within the present datasets.

Another avenue for future investigation is to assess the degree to which different learning rate groups would benefit optimally from different KC models, via KC model search (as in [13]).

6. ACKNOWLEDGMENTS

We thank Carnegie Learning, Inc. for providing the majority of the datasets that were analyzed, Christopher MacLellan for insightful discussion, IES for support to RL (training grant #R305B110003), and NSF for support to LearnLab (#SBE-0836012).

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